171 Final Exam Name: _____

1. Definition: the derivative of a function f denoted by $f'(x)$ is

   $f'(x) =$

2. Definition: $\lim_{x \to a} f(x) = L$ means

3. Definition: a function f is continuous at a number a if

4. Give an example of a function f that is continuous at $x = 1$ but NOT differentiable at $x = 1$

5. Let $f(x) = \sqrt{x+1}$ Then $f(3) =$_____ , $f'(3) =$_____

6. Find the equation of the line tangent to $y = \sqrt{x+1}$ at $x = 3$

7. Find the derivatives of the following functions. Simplify the answer if possible, but don't do anything silly.

(a) $f(x) = x^2 e^{-x}$

(b) $f(x) = \sqrt{1 - x^2}$

(c) $f(x) = \tan(x) e^{-2x}$

(d) $f(x) = \dfrac{x - 3}{x + 2}$

(e) $f(x) = 5^x$

(f) $f(x) = \log_5(x)$

(g) $f(x) = \sin^{-1}(x^2)$

(h) $f(x) = x^{\sqrt{x}}$

8. Let $f(x) = 2x^3 - 6x^2$

   (a) Find the intervals over which f is increasing and decreasing

   (b) Find the intervals over which f is concave up and concave down

   (c) Find all local max and min

9. Let $f(x) = x^2 e^{-x}$

   (a) Find the critical points of f

   (b) Find the the local max and min of f

10. Find the slope of the line tangent to the graph of $x^2 + xy + y^2 = 19$ at the point $(2, 3)$

11. Use L'Hôpital's rule to find the following limits

    (a) $\lim_{x \to 0} \dfrac{e^{3x} - 1}{\sin(x)}$

    (b) $\lim_{x \to 1} \dfrac{x^4 - 1}{x^2 - 1}$

12. State precisely what the mean value theorem says about the function $f(x) = \dfrac{x - 3}{x + 2}$ on the interval $[3, 5]$

13. If $f(2) = 4$ and $f'(x) \leq 8$ then what is the largest possible value for $f(6)$?

14. State as precisely as you can the definition of $\int_a^b f(t)dt$

15. Find the derivative of $g(x) = \int_0^x \frac{\sin(t)}{\sqrt{t^2+1}}dt$

16. Find $F(x)$ if $F'(x) = 6x^2 + 2x + 1$ and $F(1) = 5$

17. Find the general anti-derivative of $f(x) = \cos(x) + \frac{1}{x} + \frac{1}{\sqrt{1-x^2}}$

18. Evaluate $\int_0^4 \sqrt{t}dt$

19. $\displaystyle\int_0^1 \left(\frac{1}{t^2+1} + \sin(\pi t)\right) dt$

Hint: you will need a simple u-sub to find the anti-derivative of $\sin(\pi t)$

20. Find the point on the the graph of $y = x + 3$ closest to $(1, 3)$