171 Final Exam Name: _____

1. Definition: $\lim_{x \to a} f(x) = L$ means

2. Definition: the derivative of a function f denoted by $f'(x)$ is

   $f'(x) =$_____

3. Using the definition, (not the power rule) find the derivative of $f(x) = x^2 + 2x$

4. Definition: a function f is continuous at a number a if

5. Give an example of a function f that is continuous at $x = 2$ but NOT differentiable at $x = 2$

6. Let $f(x) = \sqrt{x+3}$ Then $f(6) =$_____ , $f'(6) =$_____

7. Find the equation of the line tangent to $y = \sqrt{x+3}$ at $x = 6$

8. Find the following functions. Simplify the answer if possible, but don't do anything silly.

(a) $f(x) = x^2 \sin(x)$

(b) $f(x) = \sqrt{x^2 - 2x}$

(c) $f(x) = \sqrt{x}e^x$

(d) $f(x) = \dfrac{x-2}{x+2}$

(e) $f(x) = 2^x$

(f) $f(x) = \log_2(x)$

(g) $f(x) = \sin^{-1}(e^x)$

(h) $f(x) = x^{\frac{1}{x}}$

9. Let $f(x) = 2x^3 + 3x^2 - 12x$

   (a) Find the intervals over which f is increasing and decreasing

   (b) Find the intervals over which f is concave up and concave down

   (c) Find all local max and min

10. Let $f(x) = \sqrt{x}e^x$

   (a) Find the critical points of f

   (b) Find the the local max and min of f

11. Find the slope of the line tangent to the graph of $x^2 + xy + y^2 = 13$ at the point $(-3, 4)$

12. Use L'Hôpital's rule to find the following limits

    (a) $\lim_{x \to 0} \dfrac{e^{-2x} - 1}{\sin(x)}$

    (b) $\lim_{x \to 1} \dfrac{x^5 - 1}{x^3 - 1}$

13. State precisely what the mean value theorem says about the function $f(x) = \dfrac{x-2}{x+2}$ on the interval $[0, 4]$

14. If $f(2) = 8$ and $f'(x) \leq 10$ then what is the largest possible value for $f(7)$?

15. State as precisely as you can the defintion of $\int_a^b f(t)dt$

16. Find the derivative of $g(x) = \int_0^x \frac{e^t}{\sqrt{t^2+1}}dt$

17. Find $F(x)$ if $F'(x) = 9x^2 + 3x + 1$ and $F(1) = 3$

18. Find the general anti-derivative of $f(x) = \sin(x) + \frac{1}{x} + \frac{1}{\sqrt{1-x^2}}$

19. Evaluate $\int_0^4 (x^2 + 1)dx$

20. $\int_0^2 x\sqrt{4-x^2}dx$

Hint: you will need a simple u-sub to find the anti-derivative

21. Find the point on the the graph of $y = x + 4$ closest to $(1, 4)$