In class 6

1. Factor 540 as a product of primes.
2. Factor 504 as a product of primes.
3. Find $lcm(540, 504)$
4. Use the Euclidean algorithm to find $gcd(1001, 1331)$
5. Use the Euclidean algorithm to find $gcd(277, 123)$
6. Using your answer in #5, find the Bezout coefficents of 277 and 123. That is, find s and t with $123s + 277t = 1$
7. What is the inverse of 123 modulo 277?
8. Find the inverse of 2 modulo 17.
9. Find the inverse of 34 modulo 89.
10. Solve $34x \equiv 7 (\text{mod } 89)$
11. Solve $2x \equiv 5 (\text{mod } 17)$
12. Use CRT to solve the system of congruences:
    $x \equiv 2 (\text{mod } 3), x \equiv 1 (\text{mod } 4), x \equiv 3 (\text{mod } 5)$
13. Use CRT to solve the system of congruences:
    $x \equiv 2 (\text{mod } 3), x \equiv 3 (\text{mod } 7), x \equiv 1 (\text{mod } 10)$

Euler's totient or phi function, $\phi(n)$ is an arithmetic function that counts the number of positive integers less than or equal to n that are relatively prime to n. That is, if n is a positive integer, then $\phi(n)$ is the number of integers k in the range $1 \leq k < n$ for which $gcd(n, k) = 1$

For example, $\phi(10) = 4$ as the numbers relatively prime to 10 are 1, 3, 7, and 9.

14. Find $\phi(5), \phi(12), \phi(7)$

15. Find $\phi(35), \phi(25), \phi(49)$

16. Find $\phi(p)$ for any prime p

17. Find $\phi(1225)$ Hint: $1225 = 5^2 \times 7^2$

18. Find $\phi(5^7)$ Hint, think of how many possible numbers are NOT relatively prime to 5^7 and subtract that from 5^7

19. What is $\phi(p^k)$ if p is prime and k is a positive integer?

20. Find $\sum_{d|30} \phi(d)$ that is, the sum of the totient function evaluated at all the positive divisors of 30.