163 Homework 4 Name: _____

1. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ by $x \mapsto mx + b$ i.e. a line where m is the slope and b is the y-intercept.
   (a) Show f is a bijection
   (b) Find and explicit formula for f^{-1}
2. Give an explicit formula for a function $\mathbb{Z} \longrightarrow \mathbb{N}$ that is
   (a) injective but not surjective.
   (b) surjective but not injective.
   (c) injective and surjective.
   (d) neither injective nor surjective.
3. Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ via
   $x \mapsto \lfloor \frac{x}{2} \rfloor$
   (a) Show f is surjective
   (b) Show f is not injective
4. Let $f : \mathbb{N} \to \mathbb{Z}$ via
   $x \mapsto \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ -\frac{x+1}{2} & \text{if } x \text{ is odd} \end{cases}$
   (a) Show f is injective
   (b) Show f is surjective
5. If f, g are injective prove $f \circ g$ is injective.
6. Prove $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$