162 Week 1

Notes: Making sense out of the compound interest formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} \tag{1}$$

1. You deposit \$1000 in an account earning 4% interest compounded annually. How much money would you have in 15 years?

   From the previous notes we know that to increase a number by 4%, multiply by 1.04 and to do it 15 times you would have

   $$1000 \times (1.04)^{15}$$

2. You deposit \$1000 in an account earning 4% interest compounded **monthly**. How much money would you have in 15 years?

   First of all, compounded monthly does not mean you get 4% a month. Since there are 12 months in a year you get one 12th of 4% per month: $\frac{.04}{12} = \frac{4}{1200} = \frac{1}{300}$

   Therefore in one month you will have

   $$1000 \times \left(1 + \frac{1}{300}\right)$$

   In one year:

   $$1000\left(1 + \frac{1}{300}\right)^{12}$$

   since it has been compounded 12 times.

   In 15 years, it will have been compounded $15 \times 12 = 180$ times. Therefore in 15 years the total will be

   $$1000 \times \left(1 + \frac{1}{300}\right)^{180}$$

   Wolf

3. Invest \$50 at 9% interest compounded quarterly for 10 years the total will be

   $$50 \times \left(1 + \frac{.09}{4}\right)^{40}$$

4. In general, if you invest P at a rate of r compounded n times per year, after t years you will have

   $$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} \tag{2}$$

5. It should be clear that the more frequently you compound the more money you get because you get interest on the interest (however small)

   Suppose you deposit \$1000 at 100% interest (an excellent rate of return) but only for one year.

   (a) If you compound only once per year, you will have \$2000

   (b) If you compound quarterly you will do better:

$$1000\left(1+\frac{1}{4}\right)^4 \approx 2441.41$$

   Compounded monthly gets even more

$$1000 \times \left(1+\frac{1}{12}\right)^{12} \approx 2613.04$$

   Wolf

   (c) You can compound as frequently as you like. Homework to try (no credit) What is the most money you can make?