Worksheet on rational exponents.

Recall that for a rational exponent, the numerator is the power and the denominator is the root. For example $27^{\frac{2}{3}}$ means

a) The cube root of 27 squared, or

b) The square of the cube root of 27.

a) 27 squared is $27 \times 27 = 729$ and the cube root of $729 = \sqrt[3]{729} = 9$ which you can find either using a calculator, or by recognizing that $9 \times 9 \times 9 = 729$.

b) The cube root of 27 is 3 because $3^3 = 3 \times 3 \times 3 = 27$ and then $3^2 = 9$.

In either case we see that $27^{\frac{2}{3}} = 9$, although taking the root first is usually easier if for no other reason than the numbers are smaller.
Compute the following:

1. $8^{\frac{4}{3}}$
2. $16^{\frac{1}{2}}$
3. $36^{\frac{3}{2}}$
4. $32^{\frac{2}{5}}$
5. $(144 \times 121)^{\frac{1}{2}}$
   Answer:
   $(144 \times 121)^{\frac{1}{2}} = 144^{\frac{1}{2}} \times 121^{\frac{1}{2}}$
   $= 12 \times 11 = 132$
6. $(16 \times 81)^{\frac{1}{4}}$
7. $\left(\frac{49}{64}\right)^{\frac{1}{2}}$
8. $\left(\frac{125}{8}\right)^{\frac{1}{3}}$
9. $(9 + 16)^{\frac{1}{2}}$
   Here you first add, then take the square root.
10. $9^{\frac{1}{2}} + 16^{\frac{1}{2}}$
    Here you take the square roots first, then add.
11. Which answer is bigger, the sum of the square roots, or the square root of the sum?

To annoy you further, we can also have negative exponents. This is not such a big deal if we remember that a negative exponent just means the reciprocal.

For example $\left(\frac{16}{9}\right)^{-\frac{3}{2}}$ looks bad, but if we tease out the meaning it is comprehensible. The exponent has three parts:

a) a minus sign,

b) a 2 in the denominator,

c) a 3 in the numerator.

In turn these mean a) the reciprocal, b) the square root, c) the cube (third power). Taking care of these one at a time we get

a) $\left(\frac{16}{9}\right)^{-\frac{3}{2}} = \left(\frac{9}{16}\right)^{\frac{3}{2}}$
Taking the reciprocal gets rid of the "minus" sign in the exponent.

b) $\left(\frac{9}{16}\right)^{\frac{3}{2}} = \left(\frac{3}{4}\right)^{3}$
Taking the square root gets rid of 2 in the denominator. Finally,

c) $\left(\frac{3}{4}\right)^{3} = \frac{27}{64}$
Not too bad.

Compute:

1. $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$
2. $36^{-\frac{3}{2}}$
3. $\left(\frac{49}{64}\right)^{-\frac{1}{2}}$
4. $\left(\frac{125}{8}\right)^{-\frac{1}{3}}$
5. $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$
6. $32^{-\frac{1}{5}}$