Absolute value equations and inequalities.
Some facts about absolute value:

1. If $p \geq 0$ then $|p| = p$; whereas if $p < 0, |p| = -p$
2. Geometrically, $|p|$ is the distance of p from 0.
   (a) $|-5| = 5$
   (b) $|x-3| = x-3$ if $x-3 \geq 0$ i.e. if $x \geq 3$ whereas if $x < 3, |x-3| = -(x-3) = 3-x$
3. If $|p| = b, b > 0$, then $p = b$ or $p = -b$
4. If $|p| < b, b > 0$, then $-b < p < b$
   (a) $|x| = 2$ means $x = 2$ or $x = -2$
   (b) $|x| < 2$ means $-2 < x < 2$
5. $|p - a| = b, b > 0$ means the distance of p from a is b.
6. $|p - a| \leq b, b > 0$ means the distance of p from a is less than b. Therefore p will lie in one interval:
   $-b + a < p < b + a$
7. $|p - a| > b, b > 0$ means the distance of p from a is greater than b. Therefore p may lie in two intervals:
   $p - a < -b + a$ or $p > b + a$
   (a) $|x - 3| = 4$ means x is 4 units from 3.
   (b) $|x + 1| < 3$ means x is less than 3 units from -1
   (c) $|2x + 8| > 6$ means $2x$ is greater than 6 units from -8
8. $|a - b| = |b - a|$ since it is obvious that the distance between a and b is the same as the distance between b and a.

Example 1 Solve for $x : |x - 3| = 4$
Either $x - 3 = -4$ or $x - 3 = 4$.
$x - 3 = -4 \iff x = -1$
$x - 3 = 4 \iff x = 7$
The two solutions are $\{-1, 7\}$

Example 2 Solve for $x : |x + 1| < 3$
$-3 < x + 1 < 3 \iff -4 < x < 2$. The answer is one interval:
$\{x : -4 < x < 2\}$ In interval notation $(-4, 2)$

Example 3 Solve for $x : |2x + 8| > 6$

Either $2x + 8 < -6 \iff 2x < -14 \iff x < -7$

or $2x + 8 > 6 \iff 2x > -2 \iff x > -1$

The two intervals are $\{x : x < -7\}$ or $\{x : x > -1\}$

In interval notation: $(-\infty, -7) \cup (-1, \infty)$

Note that this is two intervals, and cannot be written as one. The common mistake is to write $-1 < x < -7$ which is meaningless.

Also note that once we have solved $|x + 1| < 3$ we have also solved $|x + 1| \geq 3$ since it is everything not in $(-4, 2)$ namely $(-\infty, -4] \cup [2, \infty)$

Solve for x:

1. $|x - 2| = 5$
2. $|x + 3| = 4$
3. $|x - \pi| = 1$
4. $|x - a| = b, b > 0$
5. $|2x + 3| = 5$
6. $|ax + b| = c, c > 0$
7. $|x + 3| = -2$
8. $|x - 2| < 4$
9. $|x - 2| \geq 4$
10. $|x + 3| \leq 3$
11. $|x + 3| > 3$
12. $|2x + 1| < 7$
13. $|2x + 1| > 7$
14. $|3x - 4| < 11$
15. $|4 - 3x| < 11$
16. $|2x - 1| > -3$
17. $|2x - 1| = x + 5$
18. $|2x - 1| = |x + 5|$
19. $|2x - 6| < x + 4$ Hint: Solve the two inequalities seperately.
20. $|2x - 8| < 2x + 4$